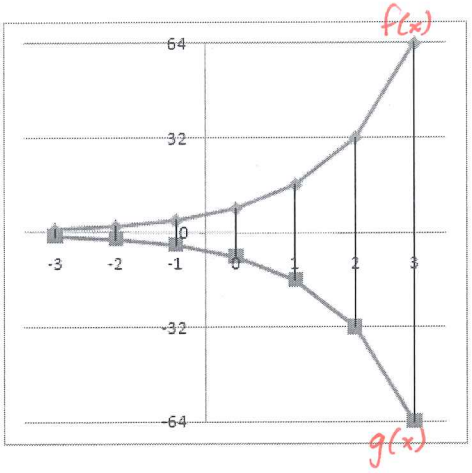
Sec. 6.2 Reflections and Symmetry

For a function f:

- The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ about the x-axis.
- The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ about the y-axis.

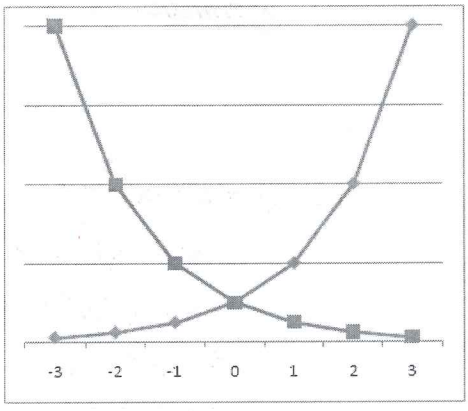
Ex. The graph shows a function $f(x)$ in blue and its reflection through the x-axis in red ($g(x)$). Create a table to show values for $f(x)$ and $g(x)$. Write a formula for $g(x)$ in terms of $f(x)$.



x	f(x)	g(x)
-3	1	-1
-2	2	-2
-1	4	-4
0	8	-8
1	16	-16
2	32	-32
3	64	-64

When a point is reflected vertically about the x-axis, the x-value stays the same while the y-value changes signs, therefore $g(x) = -f(x)$

Ex. The graph shows a function $f(x)$ in blue and its reflection through the y-axis in red ($h(x)$). Create a table that shows values for $f(x)$ and $h(x)$. Write a formula for $h(x)$ in terms of $f(x)$.

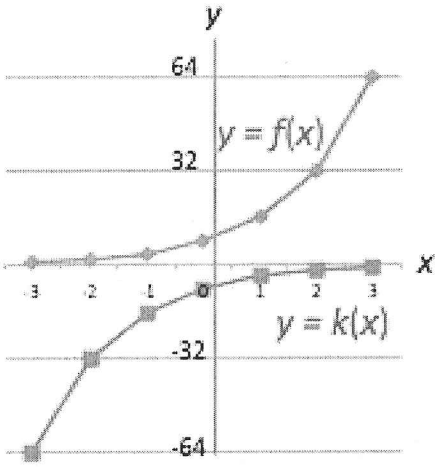


x	f(x)	h(x)
-3	1	64
-2	2	32
-1	4	16
0	8	8
1	16	4
2	32	2
3	64	1

When a point is reflected horizontally about the y-axis, the y-value remains fixed while the x-value changes signs, therefore

$h(x) = f(-x)$

Ex: The graph shows a function $f(x)$ in blue and its reflection through both the x-axis the y-axis in red ($k(x)$). Make a table shows values for $f(x)$ and $k(x)$. Write a formula for $k(x)$ in terms of $f(x)$.



x	f(x)	k(x)
-3	1	-64
-2	2	-32
-1	4	-16
0	8	-8
1	16	-4
2	32	-2
3	64	-1

When a point is reflected about the x-axis and the y-axis, both the x-value and the y-value change signs, therefore

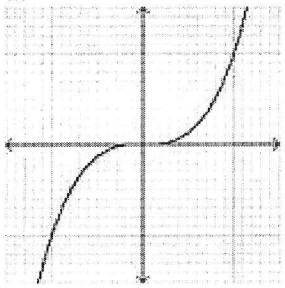
$K(x) = -f(-x)$

Even Function – A function is even if for every number x in the domain, the number –x is also in the domain and f(-x) = f(x). (Symmetric to the y-axis.)

Odd Function – A function is odd if for every number x in the domain, the number –x is also in the domain and f(-x) = - f(x). (Symmetric to the origin.)

Ex. Determine whether the following are even or odd functions. Verify using algebra when necessary.

a.



ODD SYMMETRIC TO ORIGIN

b. $f(x) = x^2 - 5$

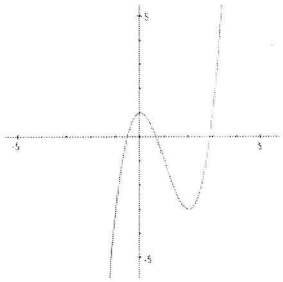
$f(-x) = (-x)^2 - 5$

$f(-x) = x^2 - 5$

$f(-x) = f(x)$

EVEN

c.



NEITHER

d. $g(x) = x^3 - 1$

$g(-x) = (-x)^3 - 1$

$(-1x)^3 - 1$

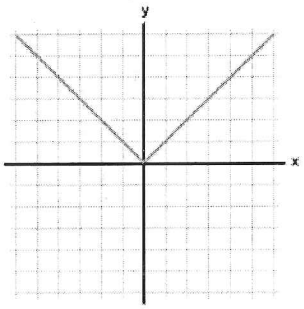
$-1x^3 - 1$

$g(-x) = -(x^3 + 1)$

$g(-x) \neq -g(x)$

NEITHER

e.



EVEN SYMMETRIC TO Y-AXIS

f. $h(x) = 5x^3 - x$

$h(-x) = 5(-x)^3 - x$

$= 5(-1x)^3 + x$

$= 5(-1)^3 x^3 + x$

$= -5x^3 + x$

$h(-x) = -(5x^3 - x)$

$h(-x) = -h(x)$

ODD

HW: pg 239-242 #3-48 (m/3)